# Engineering Notes

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## Scaling of Heat Transfer to Dilute Polymeric Solutions

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### Nomenclature

$c_p$ = specific heat
$f$ = Darcy-Weissbach friction coefficient
$k$ = thermal conductivity
$N_{Nu}$ = Nusselt number, Eq. (18)
$N_{Pr}$ = Prandtl number, $\nu/k$
$N_{Re}$ = Reynolds number, $2Ru_b/\nu$
$N_{St}$ = Stanton number, Eq. (19)
$q$ = average heat flux
$R$ = pipe radius
$R^+$ = $RV_*/\nu$
$T$ = temperature
$T_w$ = temperature at the wall
$T^+$ = $(T_w - T)c_p\rho V_*/q$
$T_b^+$ = dimensionless bulk temperature, Eq. 16
$u$ = local mean axial velocity
$u^+$ = $u/V_*$
$u_b$ = average velocity
$V_*$ = shear velocity
$y$ = distance from the wall
$y^+$ = $yV_*/\nu$
$y_1$ = edge of laminar sublayer
$y_1^+$ = $y_1V_*/\nu$
$y_2$ = edge of buffer zone
$y_2^+$ = $y_2V_*/\nu$
$\delta_1$ = edge of diffusion sublayer
$\delta_1^+$ = $\delta_1V_*/\nu$
$\delta_2$ = edge of viscous sublayer
$\delta_2^+$ = $\delta_2V_*/\nu$
$\Delta B$ = shift in the logarithmic velocity profile
$\nu$ = kinematic viscosity

THE possibility of correlation between drag reduction and changes in heat transfer is examined, possible semiempirical models for the description of these phenomena are suggested.

Drag reducing polymeric additives increase the thickness of the viscous sublayer in turbulent flow and thus reduce the friction loss in the flow. In smooth pipe flow this increase can be calculated from the equation

$$\delta_2^+ - 5.75 \log_{10}\delta_2^+ = 5.5 + \Delta B \tag{1}$$

where $\Delta B$ expresses the polymeric effects. Several semiempirical models, by which relationships between the function $\Delta B$ and the properties of the polymeric additives as well as the characteristics of the flowfield, were suggested.[1-4] However, almost none of these models can be applied for all kinds of polymers and usually their applications are limited to certain ranges of flow shear stresses.

Fabula[5] reviewed the experimental investigations concerning drag reduction and concluded that the maximum value of $\Delta B$ is smaller than 30. Virk[6] found some empirical formulation for cases of maximum drag reduction in small diameter pipes; such cases are defined through the following relationships between Reynolds number and Darcy-Weissbach friction coefficient

$$1/(f)^{1/2} = 11.5 \log_{10}[N_{Re}(f)^{1/2}] - 25 \tag{2}$$

This equation can be approximated by[7]

$$f = 1.68N_{Re}^{-0.55} \tag{3}$$

Assuming logarithmic velocity profile in the turbulent region

$$u^+ = 5.75 \log_{10}(y^+/\delta_2^+) + \delta_2^+ \tag{4}$$

and integrating the velocity profile through the whole pipe cross section, one obtains

$$\left(\frac{8}{f}\right)^{1/2} = 5.75 \log_{10} \frac{N_{Re}(f/32)^{1/2}}{\delta_2^+} + \delta_2^+ - 3.75 \tag{5}$$

By comparison of Eqs. (1) and (5) with Eq. (2), we get an expression for $\Delta B$ in cases of maximum drag reduction

$$\Delta B = 26.8 \log_{10}[N_{Re}(f)^{1/2}] - 68.5 \tag{6}$$

Assuming that $\Delta B$ should be smaller than 30, it seems that Eq. (6) can be assumed reliable while Reynolds number is smaller than $9 \times 10^4$.

The experimental results of Virk[6] in the above range of Reynolds number were revalidated by Paterson and Abernathy[8] through many experiments.

A few theoretical models which suggest correlations between heat transfer and drag reduction were reported. Poreh and Paz[9] extended von Kármán[10] semiempirical model for this purpose. According to this model the velocity profile consists of three different regions:

a) Laminar zone in which

$$u^+ = y^+ \tag{7}$$

whose thickness is $y_1^+$. In the Newtonian case $y_1^+ = 5$.

b) Buffer zone in which

$$u^+ = y_1^+ \ln(y^+/y_1^+) + y_1^+ \tag{8}$$

c) Turbulent region in which the velocity profile is expressed by Eq. (4).

By calculating heat flux it was found that according to von Kármán model

$$N_{St} = \left(\frac{f}{8}\right)^{1/2} \Bigg/ \left\{y_1^+\left\{\ln\left[N_{Pr} - (N_{Pr} - 1)\frac{y_1^+}{y_2^+}\right] + (N_{Pr} - 1)\right\} + \left(\frac{8}{f}\right)^{1/2} + \frac{125}{16}\left(\frac{f}{8}\right)\right\} \tag{9}$$

Poreh and Paz[9] assumed that in the cases of Newtonian as well as polymeric solutions

$$y_1^+/\delta_2^+ = 0.43 \tag{10}$$

Received May 25, 1971; revision received June 18, 1971. This research was sponsored by the Office of Naval Research under Contract N00014-71-C-0063.
Index Category: Boundary Layers and Convective Heat Transfer—Turbulent.

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thus they could calculate heat-transfer coefficients for the case of the flow of dilute polymeric solutions. However, their analysis was based on Elata's description[2] of the function $\Delta B$ which was found to be often inaccurate.

Wells[11] used Friend and Metzner[12] semiempirical model in order to scale heat transfer to polymeric solutions, according to this model

$$N_{St} = (f/8)/[1.02\delta_2^+(f/8)^{1/2}(N_{Pr} - 1)(N_{Pr})^{-1/3} + 1.2]$$

Wells[11] used Meyer's approach[1] for the description of the function $\Delta B$ in order to correlate this model with experimental results. However, Meyer's model was found to be unreliable in cases of high shear stresses.

Levich[13] developed a semiempirical model by which he could describe mass diffusion to Newtonian fluids for quite wide ranges of Reynolds and Schmidt numbers. Rubin[14] showed how changes in mass diffusion by polymeric additives can be predicted by using this model. However, he applied Meyer model[1] for the description of $\Delta B$; therefore his description could not be reliable for the whole range of shear stresses.

In this study it was found that Levich model can be applied for the description of heat transfer to drag reducing polymeric solutions. Comparison of the predictions of this model with experimental results showed that the reliability of this model is usually higher than that of other models used for the same purposes.

Applying Levich model for the heat-transfer problem we should assume that the flowfield consists of three different regions:

a) Diffusion sublayer, in which heat transfer as well as momentum transfer are governed by molecular actions. The dimensionless thickness of the diffusion sublayer is

$$\delta_1^+ = \delta_2^{+0.75}/N_{Pr}^{0.25} \qquad (12)$$

and the dimensionless temperature profile in this region is

$$T^+ = N_{Pr}y^+ \qquad (13)$$

b) Viscous sublayer, in which momentum transfer is still governed by molecular action; but, heat transfer is governed by the small scale turbulence which exists in this region. By applying the mixing length theory, the dimensionless temperature profile in this region is obtained

$$T^+ = N_{Pr}\delta_1^+ + (\delta_2^{+3}/3)\,(1/\delta_1^{+3} - 1/y^{+3}) \qquad (14)$$

c) The turbulent region, in which momentum transfer as well as heat transfer are governed by the turbulent fluctuations. In this region

$$T^+ = N_{Pr}\delta_1^+ + \frac{\delta_2^{+3} - \delta_1^{+3}}{3\delta_1^{+3}} + 5.75\log_{10}\left(\frac{y^+}{\delta_2^+}\right) \qquad (15)$$

the bulk dimensionless temperature is defined by

$$T_b^+ = \int_0^{R^+} T^+u^+(R^+ - y^+)dy^+ \Big/ \int_0^{R^+} u^+ \times (R^+ - y^+)dy^+ \qquad (16)$$

If the pipe diameter is large enough it is possible to neglect the diffusion and viscous sublayers effect in Eq. (16); thus the following is obtained

$$T_b^+ = N_{Pr}\delta_1^+ + \frac{\delta_2^{+3} - \delta_1^{+3}}{3\delta_1^{+3}} + \left(\frac{8}{f}\right)^{1/2} + \frac{125}{16}\left(\frac{f}{8}\right) \qquad (17)$$

Nusselt and Stanton numbers which are the heat-transfer coefficients

$$N_{Nu} = 2N_{Pr}R^+/T_b^+ \qquad (18)$$

$$N_{St} = N_{Nu}/(N_{Pr}N_{Re}) \qquad (19)$$

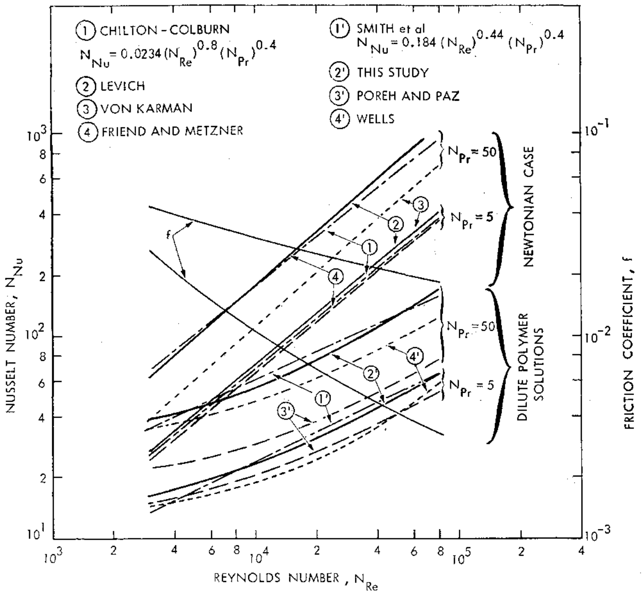


**Fig. 1 The dependence of Nusselt number and the friction coefficient upon Reynolds and Prandtl numbers.**

can be found after $T_b^+$ is calculated. In the case of dilute polymeric solutions one may assume that the viscous sublayer will be thickened more than the diffusion sublayer. Such possibility can be described by assuming that Eq. (12) is applicable for the polymeric solutions as for the Newtonian case. Another extreme possibility is that the polymeric additives only increase the thickness of the viscous sublayer with no change in the diffusion sublayer. These assumptions should be compared with experimental results. From such comparison we found that the first from the above assumptions is more realistic.

Smith et al.[7] measured the effects of drag reducing polymeric additives on heat transfer phenomenon in cases of maximum drag reduction. They found that heat-transfer reduction was limited by the following best fit asymptote

$$(N_{St})(N_{Pr})^{0.6} = 0.184(N_{Re})^{-0.54} \qquad (20)$$

This result was obtained for quite a wide range of Prandtl numbers. We may assume that comparison of the different theoretical models with the above experimental results will

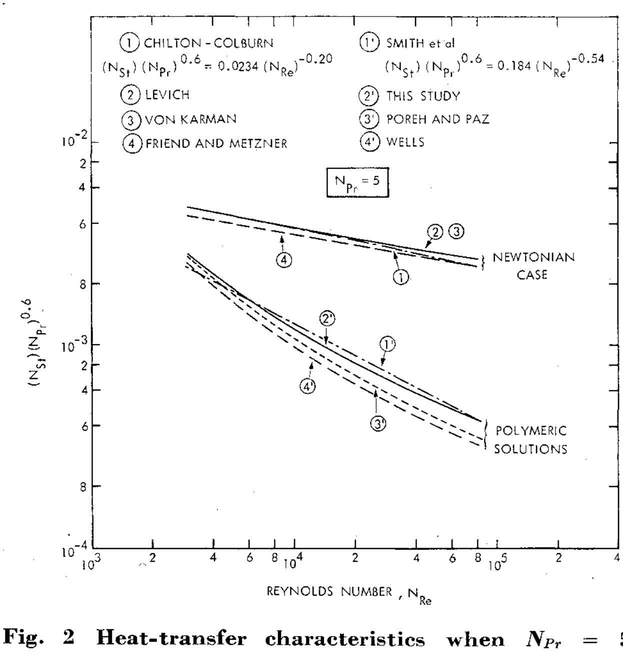


**Fig. 2 Heat-transfer characteristics when $N_{Pr} = 5$.**

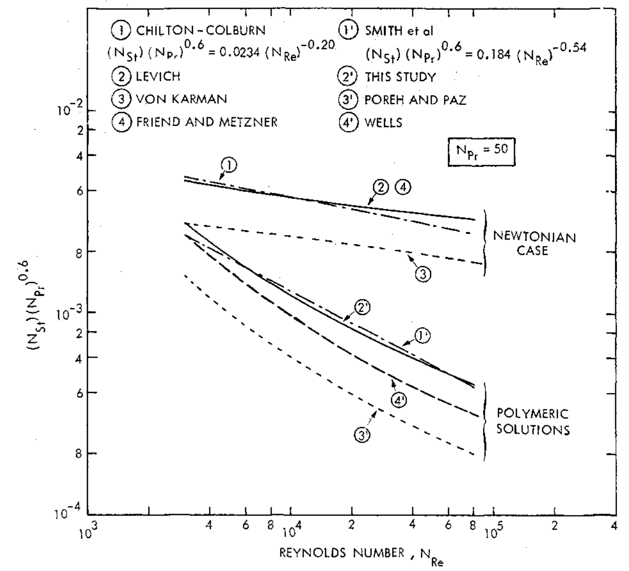


**Fig. 3 Heat-transfer characteristics when $N_{Pr} = 50$.**

give us some indication about the reliability of these models; as conditions of maximum drag reduction are well defined.

In Fig. 1 comparisons between the different models when Prandtl numbers are 5 and 50 were done. It seems that when $N_{Pr} = 5$ the model suggested in this study is closer to the experimental results than the other models; however, the differences in the values of Nusselt numbers predicted by the different models are not large. In the case of $N_{Pr} = 50$ these differences are much larger, the suggested model of this study is close to Chilton-Colburn curve as well as to Smith et al. curve. The Wells model is less close and the Poreh and Paz model is very far from both of these curves. The values of the friction coefficient in the Newtonian case as well as maximum drag reduction were shown too in this figure.

In Figs. 2 and 3 the values of $(N_{St})(N_{Pr})^{0.6}$ as a function of Reynolds number according to the different models are shown. Also from these figures it seems that the model suggested in this study can describe quite accurately heat-transfer phenomena for quite wide ranges of Prandtl as well as Reynolds numbers.

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# Drag Reduction by Use of MHD Boundary-Layer Control

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## 1. Introduction

Drag reduction for fully submerged vehicles has received considerable attention during the past decade.[1] The addition of polymer additives to the fluid in the boundary layer, preventing transition to a turbulent boundary layer by means of area or slot suction, and the optimization of vehicle shape are the methods of drag reduction that have been studied. Application of these methods has not been made, however, because of the practical problems associated with each of the proposed methods.

The method proposed here is similar to the suction method in that the boundary profile is altered to prevent transition to a turbulent boundary layer. The boundary-layer control would be effected by utilizing the Lorentz force that may be made to act on a conducting fluid particle moving in a magnetic field. An optimum magnetic field distribution will be sought to maintain a laminar boundary layer, thus achieving drag reduction and noise reduction at the same time.

Idealized problems of the motion of conducting fluids in the presence of external magnetic fields have been examined by many investigators with various inconclusive results. Hartmann and Lazarus[2] found that the transverse magnetic field was to increase the pressure gradient for the channel flow of mercury which was originally laminar, but increase of field strength produced a decrease in pressure gradient up to a certain point for the flow which was originally turbulent. Murgatroyd[3] was able to suppress turbulence at Reynolds number of $10^5$ by applying a transverse magnetic field to a channel flow of a mercury. Harris[4] extended the dimensional analysis in hydrodynamic flows to the magneto-fluid-mechanic channel flow utilizing the experimental data of Hartmann and Lazarus, and Murgatroyd. Rassow[5] studied the laminar boundary-layer flow of ionized gas over a flat plate in the presence of a transverse magnetic field and found that the skin friction was reduced but the total drag increased.

Our discussion will be mainly focused upon the submerged vehicles under seawater. The momentum integral

Received December 14, 1970; revision received May 17, 1971. The research is supported by U. S. Naval Weapons Research and Engineering Station, Newport, R. I., under Contract N00140-69-C-0338.

Index catagories: Boundary-Layer Stability and Transition; Plasma Dynamics and MHD.

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